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contributions

Calculated and Experimental Response of Thin Cylindrical Antennas to Pulse Excitation

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***Abstract*—We have computed the response of a thin, cylindrical monopole antenna to dc pulse excitation, both in transmission and reception, by use of Fourier transforms and the complex impedance and complex effective height, for pulses of space length ranging from two tenths to twice the monopole length. We have also experimentally determined the reponse and have obtained similar results. We conclude from this comparison that present steady-state antenna theory together with Fourier methods will yield reliable results in calculating the response of cylindrical antennas to dc pulses. We have also made certain other measurements of response to pulses and step functions which are reported herein.**

I. INTRODUCTION

THE CYLINDRICAL monopole and its zero-resistivity ground plane with a coaxial line connected to its terminals is shown in Fig. 1. The monopole length is designated h meter and its radius is designated a meter. The input impedance of the antenna is designated $Z_0(f)$ ohm. The effective height of the antenna is designated $h_e(f)$ meter.

In transmission, the coaxial line and its driving generator can be represented by an equivalent source of open-circuit voltage $v_g(t)$ volt and internal impedance $Z_g(f)$ ohm connected directly to the antenna terminals (see Fig. 2). We will show the time history $e^{\mathrm{rad}}(t)$ V/m of the radiation field along the ground plane (where $\theta = \pi/2$—see Fig. 1) at distance r meter from the antenna when the equivalent-source voltage $v_g(t)$ volt is a pulse.

In reception, the line and its terminating impedance can be replaced at the antenna terminals by an equivalent load $Z_L(f)$ ohm (see Fig. 2). We will show the time history of the voltage $v_L(t)$ volt across the load $Z_L(f)$ at the antenna terminals when the antenna is excited by a vertically polarized plane wave $e^{\mathrm{inc}}(t)$ V/m of pulse form.

The excitation-pulse shapes used for both transmission and reception are shown in Fig. 3. The parameters t_1 seconds and T seconds are proportional to the pulse-rise time and pulse width, respectively. In Fig. 3,

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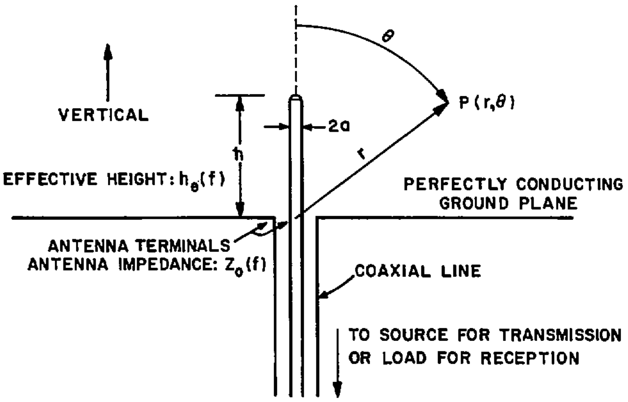
VERTICAL
θ
P(r,θ)
2a
r
h
EFFECTIVE HEIGHT: $h_e(f)$
PERFECTLY CONDUCTING GROUND PLANE
ANTENNA TERMINALS
ANTENNA IMPEDANCE: $Z_0(f)$
COAXIAL LINE
TO SOURCE FOR TRANSMISSION OR LOAD FOR RECEPTION

Fig. 1. Cylindrical monopole fed by a coaxial line.

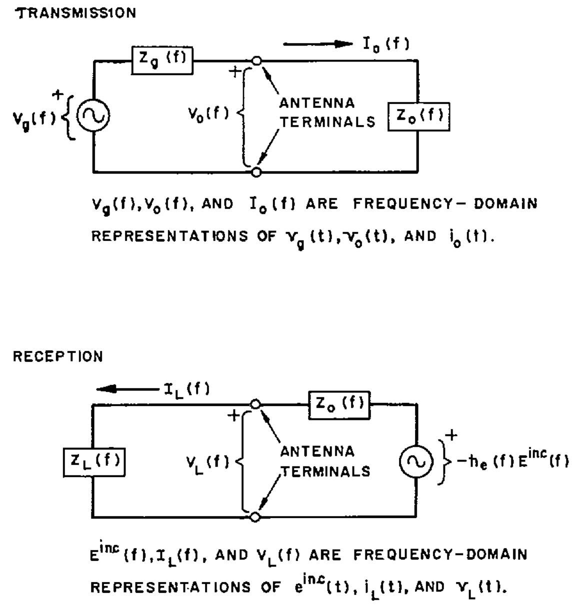
TRANSMISSION
$Z_g(f)$
$I_0(f)$
$V_g(f)$
$V_0(f)$
ANTENNA TERMINALS
$Z_0(f)$
$V_g(f), V_0(f)$, AND $I_0(f)$ ARE FREQUENCY-DOMAIN REPRESENTATIONS OF $v_g(t), v_0(t)$, AND $i_0(t)$.
RECEPTION
$I_L(f)$
$Z_0(f)$
$Z_L(f)$
$V_L(f)$
ANTENNA TERMINALS
$-h_e(f)E^{inc}(f)$
$E^{inc}(f), I_L(f)$, AND $V_L(f)$ ARE FREQUENCY-DOMAIN REPRESENTATIONS OF $e^{inc}(t), i_L(t)$, AND $v_L(t)$.

Fig. 2. Equivalent circuits for transmission and reception.

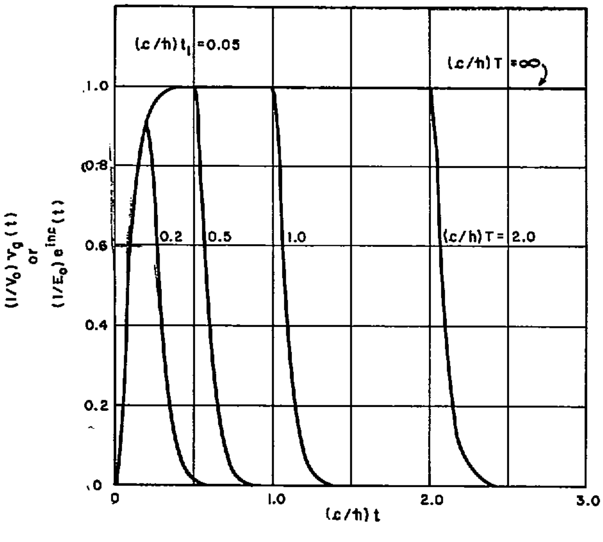

Fig. 3. The excitation-pulse shapes used in calculations and approximated in experiments. All time expressions are given with respect to the one-way travel time h/c seconds on the antenna.

all time expressions are given in units of h/c seconds, the one-way travel time along the monopole antenna.

In transmission and reception, we show the output time history both as determined by steady-state antenna theory plus Fourier methods and as measured by laboratory techniques. The purpose of giving both is to demonstrate that calculations for pulse applications using the aforementioned methods give results that are meaningful.

Since the lobe structure of a transmitting antenna changes as the frequency of the excitation voltage applied to the terminals changes, it follows that the transfer function that relates the excitation voltage to the radiated field at a given point in space will vary with frequency. The effect on pulse transmission is that the spectrum of the pulse transmitted to the given point has nulls where the spectrum of the driving pulse may not. This results, of course, in distortion of the original pulse time history. Evidently, a similar situation exists for pulse reception since reciprocity applies.

II. PRÉCIS

A. Transmission

The pulse $v_g(t)$ volt of the equivalent source connected to the antenna terminals is initiated at $t=0$, as shown in Fig. 3. Figure 4 shows the resultant calculated radiation field $e^{\mathrm{rad}}(t)$ V/m along the ground plane, where $\theta=\pi/2$, for four values of relative pulse width $(c/h)T$ (the shortest being a pulse whose space length is 0.2 of the monopole length and the longest being of space length twice the monopole length), for a relative rise time $(c/h)t_1$ of 0.05, for an antenna length-to-diameter ratio[1] h/a of 904, and for an equivalent-source impedance $Z_g(f)$ constant at 50 ohms. ($c=3\times10^8$ m/s is the velocity of electromagnetic propagation in vacuo.) The ordinate in Fig. 4 is actually $(r/V_0)e^{\mathrm{rad}}(t+r/c)$, i.e., the amplitude is multiplied by the distance r meter to remove the effect of distance on amplitude and is divided by the constant V_0 volt to remove the effect of input pulse amplitude on field amplitude, and the expression has been translated r/c seconds to the left to eliminate the delay due to travel time. The expression is plotted against $(c/h)t$, i.e., the time is expressed in units of h/c seconds.

Figure 5 shows $e^{\mathrm{rad}}(t)$ V/m as determined experimentally. The agreement with the calculated response of Fig. 4 is generally good, although the second and fourth pulses in Fig. 5(a) seem to be displaced to the right of the second and fourth calculated pulses in the upper trace of Fig. 4 about 0.7 ns. Moreover, the amplitude of the second pulse with respect to the first is 1.72 in the calculations and is about 1.1 in the experiment; this ratio is about 0.65. Let us consider the geometry of the experiment shown in Fig. 6 in order to find an explanation of these discrepancies.

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[1] For $h/a=904$, $\Omega=2\ln(2h/a)=15.0$.

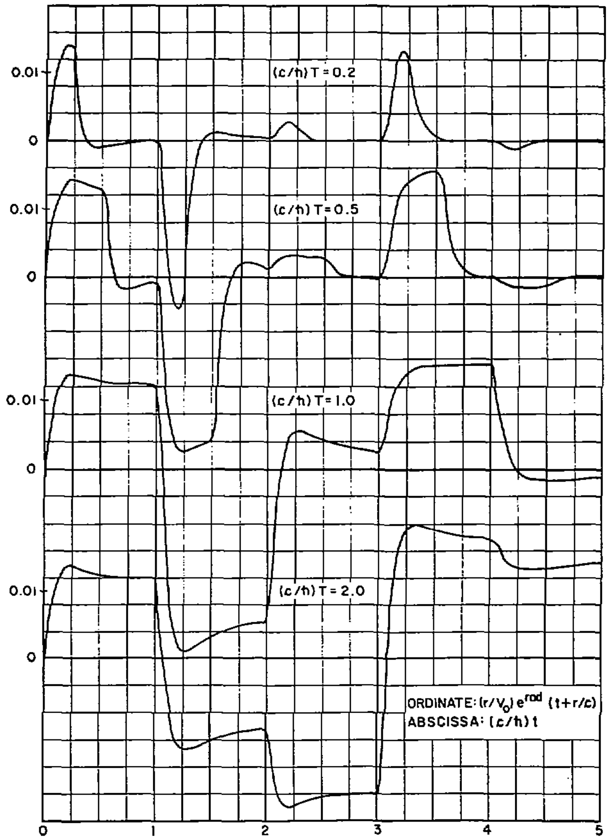
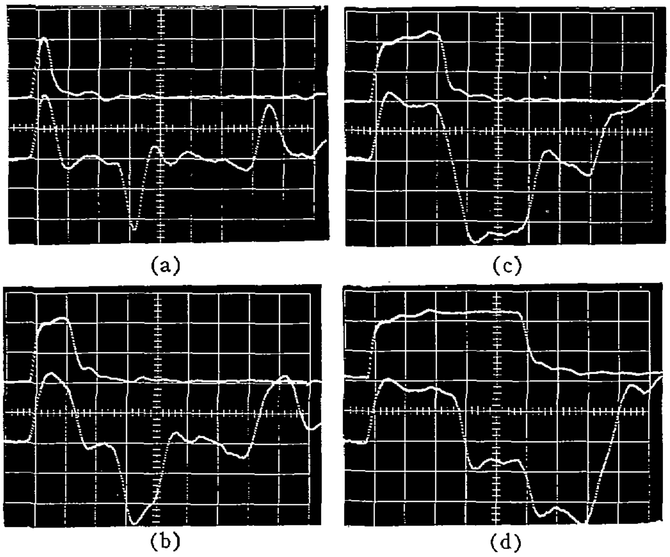


Fig. 4. The time history of the calculated radiation field $e^{\rm rad}(t+r/c)$ V/m along the ground plane for the excitation $v_g(t)$ volt shown in Fig. 3 for $Z_g(f)=Z_g=50$ ohms, for $h/a=904$, for $(c/h)t_1=0.05$, and for several values of $(c/h)T$.

The first pulse radiated after single-pulse excitation arises as the charge flows onto the antenna from the source; the second radiated pulse is due to the reversal of charge flow at the end of the antenna; the third is caused by the second reversal at the impedance discontinuity between antenna and its source; etc. [1]. Thus, the second, fourth, etc., radiation pulses in each trace have their source at the free end of the monopole while the first, third, etc., originate at the junction of monopole and driving source. Now the computations were based on the assumption that the point $(r, \pi/2)$ where the field was calculated was so far from the antenna that the distances to the point from either end of the antenna were essentially equal. If we examine Fig. 6, we note that this is far from true in the experiment. In fact, the two distances differ by 0.22 meter which represents a difference in travel time of 0.74 ns. This accounts quite well for the rightward displacement of the second and fourth pulses in Fig. 5.

The amplitude of the second pulse with respect to the first is affected by three factors:

1) It travels a distance that is greater by a factor of 1.14 (see Fig. 6); since the field varies as $1/r$, this reduces the second pulse by the factor of 1/1.14.


(a) (c)

(b) (d)

Fig. 5. The time history of the measured radiation field $e^{\rm rad}(t)$ V/m along the ground plane (lower traces) for the excitation $v_g(t)$ shown in the upper traces for $Z_g(f)=Z_g=50$ ohms, for $h/a=904$, for $(c/h)t_1=0.05$, and for $h=0.85$ m. Time scale: 1.25 ns/division. (a) $(c/h)T=0.2$, (b) $(c/h)T=0.5$, (c) $(c/h)T=1.0$, (d) $(c/h)T=2.0$.

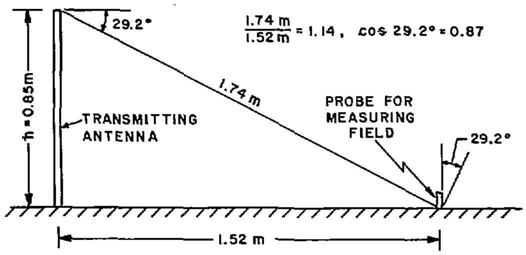

29.2°
$\frac{1.74\ \rm m}{1.52\ \rm m}=1.14$, cos 29.2° = 0.87
1.74 m
h = 0.85 m
TRANSMITTING ANTENNA
PROBE FOR MEASURING FIELD
29.2°
1.52 m

Fig. 6. Geometry of the transmission experiment.

2) The pulse leaving the end of the antenna in the direction of the probe travels in a direction that is 29.2° off the normal (see Fig. 6); this provides a factor of cos 29.2° = 0.87.
3) The effective field at the probe is also reduced by the factor cos 29.2° = 0.87 because the probe is not normal to the direction of propagation (see Fig. 6).[2]

The amplitude of the second radiation pulse as measured in the experiment should, therefore, be (1/1.14)(0.87)(0.87) = 0.66 of the calculated value. This closely accounts, then, for the lessened amplitude of the second pulse in Fig. 5.

The amplitudes of the pulses beyond the second in each trace cannot readily be precisely explained although the general agreement between experiment and calculation is good. In these later pulses, the spreading perturbations of the trailing edge of the driving pulse

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[2] The probe is very short so that the wavefront excites it throughout its length almost simultaneously. Hence, the factor cos 29.2° may be used as a first approximation.

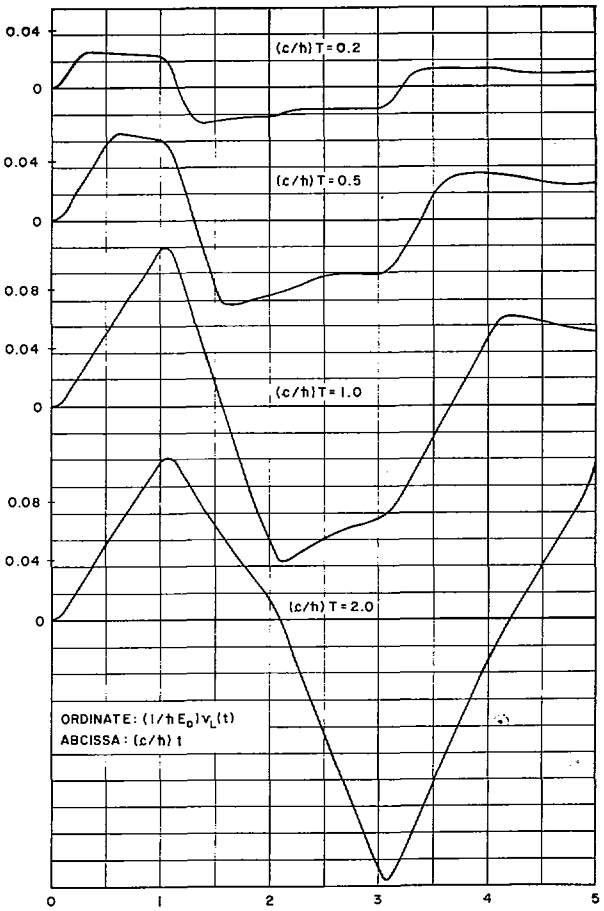

Fig. 7. The time history of the calculated voltage $v_L(t)$ volt at the antenna terminals for the incident field $e^{\mathrm{inc}}(t)$ V/m shown in Fig. 3 for $Z_L(f)=Z_L=50$ ohms, for $h/a=904$, for $(c/h)t_1=0.05$, and for several values of $(c/h)T$.

(see upper traces in Fig. 5) are beginning to affect the amplitudes.

B. Reception

The initial rise of the pulse $e^{\mathrm{inc}}(t)$ V/m of the vertically polarized incident field arrives at the antenna axis at $t=0$. Figure 7 shows the resultant calculated voltage $v_L(t)$, divided by the monopole length h meter and the constant E_0 V/m, across the equivalent-load impedance $Z_L(f)$ connected directly across the antenna terminals for four values of relative pulse width $(c/h)T$, for a relative rise time $(c/h)t_1$ of 0.05, for a length-to-diameter ratio h/a of 904, and for an equivalent-load impedance $Z_L(f)$ constant at 50 ohms.

Figure 8 shows $v_L(t)$ volt as measured experimentally. As in transmission, the agreement with the calculations is generally good. Also, as in transmission, the geometry of the experiment did not permit it to meet the assumptions used in the calculations (see Fig. 9). Specifically, the incident field emanated from the base of the generating antenna whose distance from the receiving antenna was too short to provide a plane wave there.

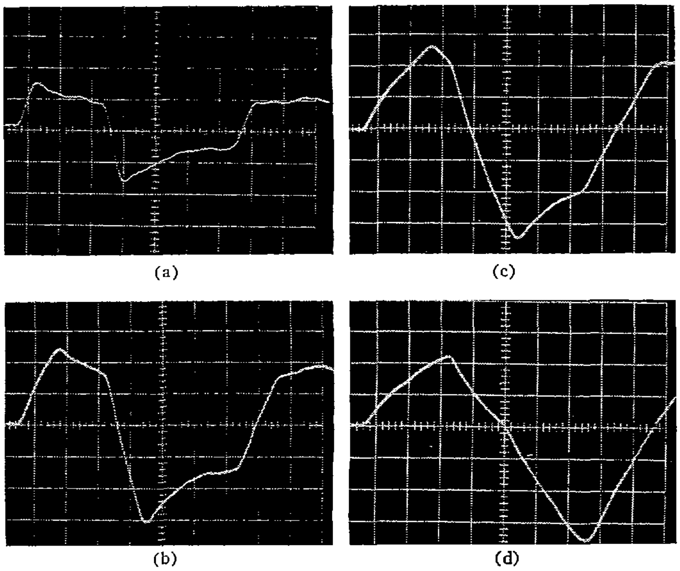

Fig. 8. The time history of the measured voltage $v_L(t)$ volt at the monopole antenna terminals for the incident field $e^{\mathrm{inc}}(t)$ shown in the upper traces of Fig. 5, for $Z_L(f)=Z_L=50$ ohms, for $h/a=904$, for $(c/h)t_1=0.05$, and for $h=0.85$ m. Time scale: 125 ns/division. (a) $(c/h)T=0.2$, (b) $T=0.5$, (c) $(c/h)T=1.0$, (d) $(c/h)T=2.0$.

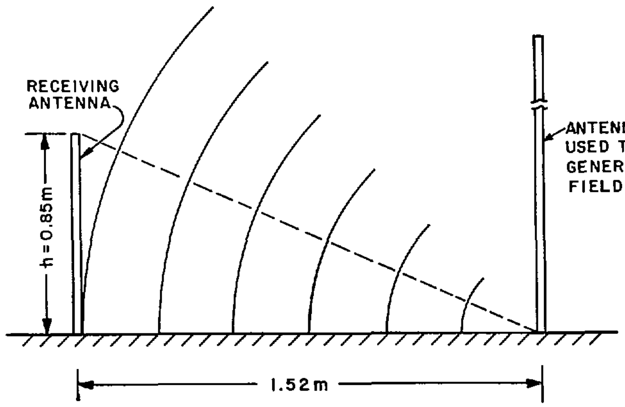

Fig. 9. Geometry of the reception experiment.

III. Analysis

A. Excitation Pulse

The pulse source used in transmission for calculating the radiation field has the voltage

$$v_g(t) = 0 \text{ volt} \qquad \text{for } t \le 0,$$

$$v_g(t) = V_0[1-(1+t/t_1)\epsilon^{-t/t_1}]\text{volt} \quad \text{for } 0 \le t \le T,$$

$$v_g(t) = V_0\left[\{1-(1+t/t_1)\epsilon^{-t/t_1}\} - \left\{1-\left(1+\frac{t-T}{t_1}\right)\epsilon^{-(t-T)/t_1}\right\}\right]\text{volt} \quad \text{for } T \le t, \qquad (1)$$

where T seconds is proportional to pulse duration, t_1 seconds is proportional to pulse-rise time,[3] and V_0 volt is a constant. The voltage $v_g(t)$, divided by the constant V_0 volt, is plotted in Fig. 3, for $(c/h)T$ with the values 0.2, 0.5, 1.0, 2.0, and ∞, and for $(c/h)t_1$ with the value 0.05. (The term h/c seconds is the one-way travel time along the antenna; all time expressions are given in units of h/c.) The frequency-domain representation of the pulse is its Fourier transform:

$$V_g(f) \equiv \int_{-\infty}^{\infty} v_g(t)\epsilon^{-j2\pi f t}dt$$

$$= \frac{V_0(1-\epsilon^{-j2\pi fT})}{j2\pi f(1+j2\pi f t_1)^2} \text{ volt/Hz.} \qquad (2)$$

In reception, the time history of the plane-wave incident field is assumed to have the same shape as that of the driving pulse in transmission, i.e.,

$$e^{\text{inc}}(t) = \frac{E_0}{V_0} v_g(t) \text{ V/m,} \qquad (3)$$

(where E_0 V/m is a constant related to the amplitude of the field pulse), which has the frequency-domain representation

$$E^{\text{inc}}(f) = \frac{E_0}{V_0} V_g(f) \text{ (V/m)/Hz.} \qquad (4)$$

The shapes of the pulses used in the transmission experiments for $v_g(t)$ are shown in Fig. 5 as the upper trace of each photograph. Clearly, these experimental pulse shapes are close to those given by (1) and plotted in Fig. 3. The shapes of the incident-field pulses used in the reception experiments are not illustrated but were similar to those used in transmission.

B. Radiation Field

The frequency-domain representation of the radiation field at distance r from the antenna (with $r \gg h$) and along the ground plane where $\theta = \pi/2$ is [2]

$$E^{\text{rad}}(f) = j\frac{\zeta_0}{2\pi}\frac{V_g(f)}{Z_0(f)+Z_g(f)}\frac{\epsilon^{-j\beta r}}{r}\beta h_e(f) \text{ (V/m)/Hz,} \qquad (5)$$

where $\zeta_0 = 120\pi$ is the characteristic resistance of space and $\beta = 2\pi/\lambda = 2\pi f/c$ is the radian wavenumber. Then, the time history of the radiation field at the same point is

$$e^{\text{rad}}(t) = \int_{-\infty}^{\infty} E^{\text{rad}}(f)\epsilon^{j2\pi f t}df \text{ V/m.} \qquad (6)$$

If $Z_g(f)$ is a constant Z_g and the excitation $V_g(t)$ is that given by (1), it can be shown that $(r/V_0)e^{\text{rad}}(t+r/c)$ depends only on $(c/h)t$, $(c/h)T$, $(c/h)t_1$, h/a, and Z_g. We have used (5) for $E^{\text{rad}}(f)$, (2) for $V_g(f)$, have taken $h/a = 904$, $Z_g = 50$ ohms, $(c/h)t_1 = 0.05$, and have given $(c/h)T$ four different values to obtain the results shown in Fig. 4 by numerical integration of (6). The input impedance $Z_0(f)$ and the effective height $h_e(f)$ for a vertically polarized incident plane wave were taken from Harrison [3], [4].

C. Load Voltage in Reception

From Fig. 2, the frequency-domain representation of the load voltage is

$$V_L(f) = \frac{-h_e(f)E^{\text{inc}}(f)Z_L(f)}{Z_0(f)+Z_L(f)} \text{ V/Hz.} \qquad (7)$$

Hence, the time history of the load voltage is

$$v_L(t) = \int_{-\infty}^{\infty} V_L(f)\epsilon^{j2\pi f t}df \text{ volt.} \qquad (8)$$

If $Z_L(f)$ is a constant Z_L and the excitation $e(t)^{\text{inc}}$ is that given by (3), it can be shown that $(1/hE_0^{\text{inc}})v_L(t)$ depends only on $(c/h)t$, $(c/h)T$, $(c/h)t_1$, h/a, and Z_L. We have used (7) for $V_L(f)$, (4) for $E^{\text{inc}}(f)$, have taken $h/a = 904$, $Z_L = 50$ ohms, $(c/h)t_1 = 0.05$, and have given $(c/h)T$ four different values to obtain the results shown in Fig. 7 by numerical integration of (8).

D. Short Antenna

For a short monopole antenna where $\beta h \ll 1$, the input impedance and effective height are [5]

$$Z_0(f) \doteq -j\frac{\zeta_0}{4\pi\beta h}(\Omega - 2 - \ln 4) \text{ ohm,} \qquad (9)$$

and [6]

$$h_e(f) \doteq \frac{h(\Omega - 1)}{2(\Omega - 2 + \ln 4)} \text{ meter,} \qquad (10)$$

where

$$\Omega \equiv 2 \ln (2h/a). \qquad (11)$$

We make the assumption that

$$|Z_g(f)| \ll |Z_0(f)|; \qquad (12)$$

this is a reasonable assumption for small βh, as (9) makes clear, provided $Z_g(f)$ is not capacitive. Then, to find the radiation field of a short cylindrical monopole antenna we substitute (9) and (10) into (5) and also use (12) to obtain[4]

$$E^{\text{rad}}(f) \doteq \frac{\epsilon^{-j\beta r}}{r}(h/c)^2(j2\pi f)^2 V_g(f) \quad \text{(V/m)/Hz.}^5 \qquad (13)$$

The term $\epsilon^{-j\beta r} = \epsilon^{-j2\pi(r/c)f}$ accounts for the travel time r/c seconds from the antenna to the point $(r, \pi/2)$; and multiplication by $j2\pi f$ in the frequency domain is equivalent to time differentiation in the time domain.

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[3] The rise time is approximately $5t_1$.

[4] We assume that $\Omega \geq 7$ since the present theory of cylindrical antennas is not valid for smaller values of Ω.

[5] The term $(\Omega-1)/(\Omega-2-\ln 4)(\Omega-2+\ln 4)$ should appear in the expressions for $E^{\text{rad}}(f)$, $e^{\text{rad}}(t)$, $V_L(f)$, and $v_L(t)$ for short antennas. However, since the term does not differ greatly from unity for $\Omega \geq 7$, we have omitted it in (13), (14), (16), and (17).

Hence, for the short antenna which satisfies $\beta h \ll 1$ and $|Z_g(f)| \ll |Z_0(f)|$,

$$re^{\mathrm{rad}}(t + r/c) \doteq (h/c)^2 \frac{d^2 v_g(t)}{dt^2} \text{ volt.[5]} \qquad (14)$$

Figure 10 shows $e^{\mathrm{rad}}(t)$ for three different pulses of $v_g(t)$ for a case where the conditions $\beta h \ll 1$ and $|Z_g(f)| \ll |Z_0(f)|$ are reasonably well satisfied for all significant frequencies in $v_g(t)$. The double differentiation is evident.

For the conditions

$$\begin{aligned} Z_L(f) &= Z_L, \quad \text{a constant,} \\ &\ll |Z_0(f)|, \end{aligned} \qquad (15)$$

we find the load voltage $V_L(f)$ of a short monopole excited by a vertically polarized plane wave $E^{\mathrm{inc}}(f)$ to be

$$V_L(f) \doteq -j2\pi f \frac{2\pi h^2 Z_L}{c\zeta_0} E^{\mathrm{inc}}(f) \text{ V/Hz,[5]} \qquad (16)$$

by use of (9), (10), and (15) in (7). Hence, the time history of the load voltage for the short antenna which satisfies $\beta h \ll 1$ and $Z_L \ll |Z_0(f)|$ (where $Z_L = Z_L(f)$, a constant), is

$$v_L(t) = -\frac{2\pi h^2 Z_L}{c\zeta_0} \frac{de^{\mathrm{inc}}(t)}{dt} \text{ volt.[5]} \qquad (17)$$

Figure 11 shows $v_L(t)$—lower traces—for two incident-field pulses—upper traces—for an antenna and load which meet the conditions $\beta h \ll 1$ and $Z_L \ll |Z_0(f)|$. The differentiation is clearly illustrated.

For a purely capacitive load

$$Z_L(f) = 1/j2\pi f C, \qquad (18)$$

which will not satisfy the conditions of (15), we find by substituting (9), (10), and (18) into (7), that

$$V_L(f) = -\frac{\alpha h/2}{1 + C/C_A} E^{\mathrm{inc}}(f) \text{ V/Hz,[6]} \qquad (19)$$

where

$$C_A \equiv \frac{4\pi h}{c\zeta_0(\Omega - 2 - \ln 4)} \text{ farad,} \qquad (20)$$

$$\alpha \equiv \frac{\Omega - 1}{\Omega - 2 + \ln 4},\text{[6]} \qquad (21)$$

and Ω is defined by (11). The time history of the load voltage for the short antenna where $\beta h \ll 1$ and $Z_L(f) = 1/j2\pi f C$ is, therefore,

$$v_L(t) = \frac{-\alpha h/2}{1 + C/C_A} e^{\mathrm{inc}}(t) \text{ volt.} \qquad (22)$$

Figure 12(c) represents a case where the conditions required for (22) to be valid are fairly well satisfied; as a result, the step-function of incident field is quite well

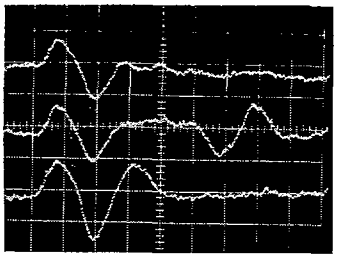


Fig. 10. The measured radiation field $e^{\mathrm{rad}}(t)$ V/m along the ground plane for the excitation $v_g(t)$ volt given by (1) for $t_1 = 0.14$ ns, for $T = \infty$, 2.5 ns, and 1 ns (upper, middle, and lower traces, respectively), for $Z_g(f) = Z_g = 50$ ohms, for $h = 0.03$ m, and $a = 1$ mm. Time scale: 0.5 ns/division.

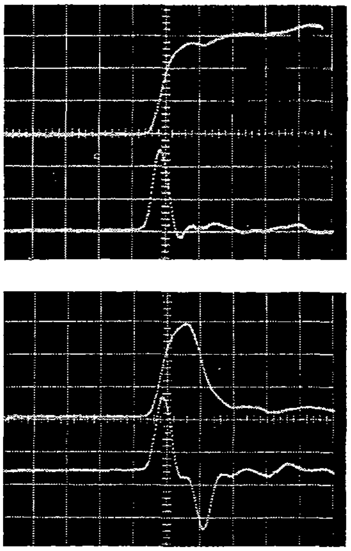
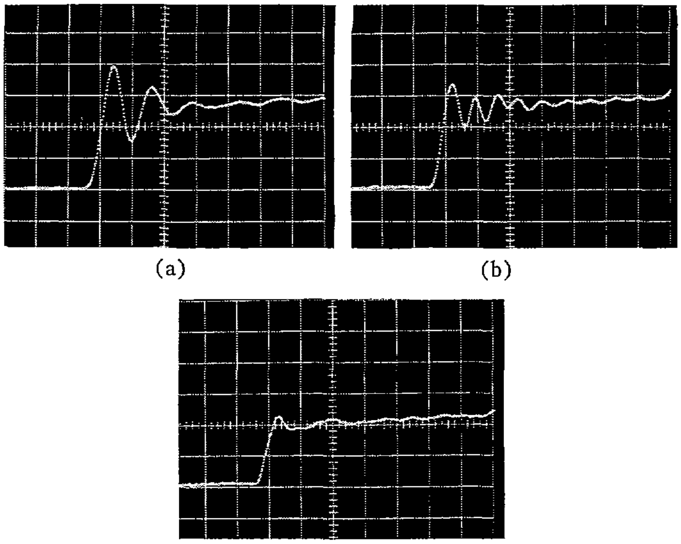


Fig. 11. The measured voltage at the antenna terminals $v_L(t)$ volt—lower traces—for the incident field $e^{\mathrm{inc}}(t)$ V/m shown in the upper traces for $Z_L(f) = Z_L = 50$ ohms for $h = 0.03$ m and $a = 1$ mm. Time scale: 0.5 ns/division.



(a) (b)

(c)

Fig. 12. Terminal voltage $v_L(t)$ of monopole with load $Z_L(f) = 1/j2\pi f C$ where $C = 3$ $\mu\mu$f and $a = 1$ mm, for (a) $h = 10$ cm, (b) $h = 5$ cm, and (c) $h = 1$ cm, for a step-function incident field $e^{\mathrm{inc}}(t)$ V/m given by (1) with $T = \infty$ and $t_1 = 0.14$ ns. Time scale: 1 ns/division.

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[6] For $\Omega \geq 7$, α does not differ greatly from unity.

reproduced at the antenna terminals. Figures 12(a) and (b) represent cases where the condition $\beta h \ll 1$ for all significant frequencies in the step-function of incident field is *not* satisfied and, as a result, the step-function is not well reproduced at the antenna terminals.

IV. MISCELLANEOUS

Let the load impedance at the antenna terminals be a resistor R and capacitor C in series:

$$Z_L(f) = R + 1/j2\pi fC.$$

The transfer function that relates the voltage $V_C(f)$ V/Hz across the capacitor C farad, divided by the monopole length h meter, to the incident field $E^{\text{inc}}(f)$ (V/m)/Hz may be derived from (7) and is

$$\frac{(1/h)V_C(f)}{E^{\text{inc}}(f)} = \frac{(1/h)V_L(f)}{E^{\text{inc}}(f)} \cdot \frac{1/j2\pi fC}{R + 1/j2\pi fC}$$
$$= \frac{-h_e(f)}{h[1 + j2\pi fRC + j2\pi fCZ_0(f)]},$$

a function of f, h/a and h, as well as of R and C. Figure 13 shows both the absolute value and argument of this transfer function for $C = 3$ $\mu\mu$f, for R ranging from 0 to 3300 ohms, for h with the fixed value 0.05 meter, and for h/a with the value 74.2 (i.e., with $\Omega = 10$). We note that, for $R = 0$ and for frequencies which satisfy the inequality $\beta h = (2\pi h/c)f \ll 1$, the transfer function is independent of frequency, as implied by (19). We also note for $R = 0$ that the resonant peak of the transfer function corresponds to a monopole length of about one-quarter wavelength. Figure 12 shows that the 1-cm monopole has its resonant frequency high enough to be beyond the significant spectral components of the incident field, but the 5-cm and 10-cm antennas do not. In fact, the spectral components at the resonant frequencies of the 5-cm and 10-cm antennas cause ringing at those frequencies.

We also observe in Fig. 13 that the addition of a resistor in series with the capacitor extends the flat portion of the transfer function to higher frequencies. However, if the resistor is too large, the transfer function drops below its dc value at a quite low frequency. Figure 14(a) shows that the addition of a 330-ohm resistor causes the capacitor voltage $v_C(t)$ of the 5-cm monopole to reproduce the step-function better than it does with no resistance as in Fig. 12(b). In Fig. 14(b), where the series resistance is 3300 ohms, the high-frequency components of the step-function that permit its rapid rise are eliminated so that the voltage at the antenna terminals rises slowly to its final value.

Figure 14(c) shows the result of using a string of five small half-watt resistors for the antenna with their resistances increasing through the values shown in the figure toward the free end of the monopole, and with a capacitive load $Z_L(f) = 1/j2\pi fC$, $C = 3$ $\mu\mu$f, at the antenna terminals.

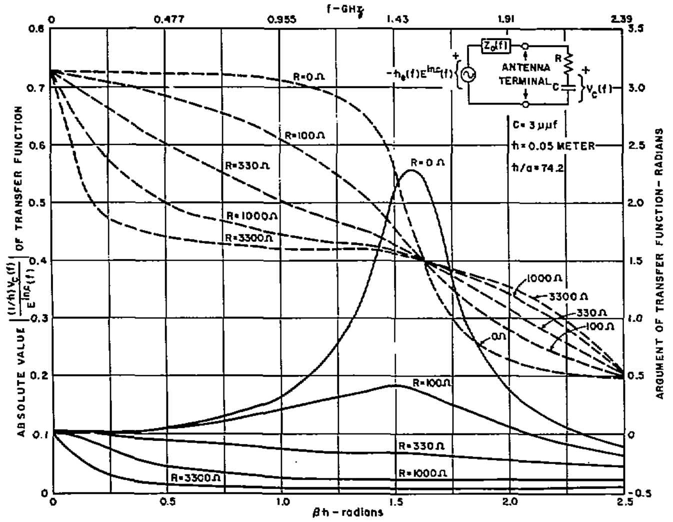
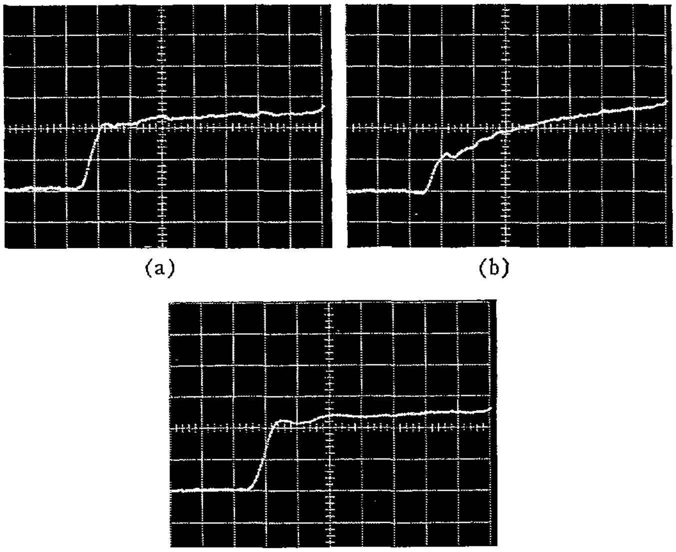


Fig. 13. Transfer function relating the voltage $V_C(f)$ V/*Hz* on the capacitor C farad, divided by the monopole length h m, to the incident field $E^{\text{inc}}(f)$ (V/m)/*Hz* for a receiving monopole with $h/a = 74.2$, $h = 0.05$ m, and $Z_L(f) = R + 1/j2\pi fC$ where $C = 3$ $\mu\mu$f.

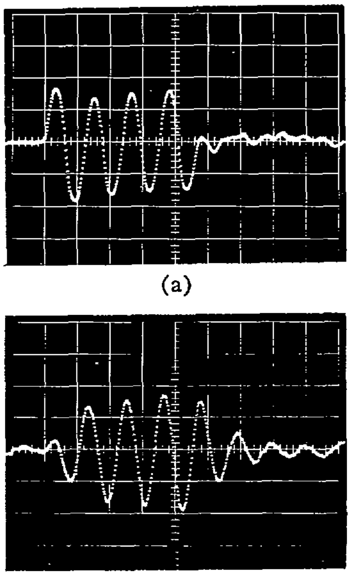

(a) (b)

(c)

Fig. 14. Capacitor voltage, $v_C(t)$ for monopole with load $Z_L(f) = R + 1/j2\pi fC$ where $C = 3$ $\mu\mu$f and (a) $R = 330$ ohms and (b) $R = 3300$ ohms for $h = 5$ cm and $a = 1$ mm. In (c), the antenna consisted of five small half-watt resistors 33, 75, 150, 330, and 750 ohms with values increasing toward the free end and with the load $Z_L(f) = 1/j2\pi fC$ where $C = 3$ $\mu\mu$f. Time scale: 1 ns/division.

Figure 15(b) shows the radiation field along the ground plane $e^{\text{rad}}(t)$ when the antenna is excited by an RF pulse $v_g(t)$ shown in Figure 15(a) for the case where the antenna is tuned to the radio frequency f_0, i.e., for $h = \lambda_0/4$ where $\lambda_0 = c/f_0$.

Figure 16 shows the radiation field along the ground plane $e^{\text{rad}}(t)$ of an antenna made of a string of resistors of tapered values. The upper trace is the exciting voltage $v_g(t)$ and the lower trace is the field $e^{\text{rad}}(t)$.



(a)

(b)

Fig. 15. The radiation field $e^{\mathrm{rad}}(t)$ V/m along ground plane (b) for $v_g(t)$ shown in (a) for a tuned monopole length $h=\lambda_0/4$ where $\lambda_0=c/f_0$ is the wavelength of the radio frequency f_0 of the pulse, for $Z_g=50$ ohms, for $h=17$ cm, and for $a=0.5$ mm. Time scale: 2 ns/division.

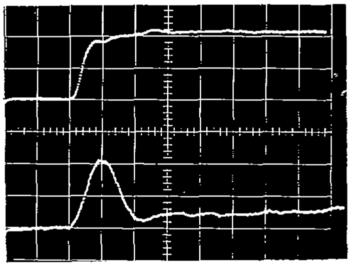


Fig. 16. The radiation field $e^{\mathrm{rad}}(t)$ V/m along ground plane (lower trace) for $v_g(t)$ shown in upper trace for a monopole made of 30 half-watt carbon resistors increasing toward the free end in steps of 10 ohms from 10 to 300 ohms and for $Z_g(f)=Z_g=50$ ohms.

V. EXPERIMENTAL SYSTEM

The experiments were performed indoors on a large ground screen with dimensions 10 feet by 12 feet and having provisions for mounting antennas and field probes to monitor the radiation field at various places on the plane. Pulses of variable length and step functions of about 0.2-ns rise time were obtained from a mercury-switch pulser. The pulses and steps were fed to the transmitting antenna through a 20-foot section of RG9U cable. The length of cable insured that no secondary response due to reflections at cable junctions could arise in a time interval of about 60 ns.

The geometry of the transmission and reception experiments is shown in Figs. 6 and 9. In the former case, as previously noted, the probe for measuring the radiation field was not far enough away from the transmitting antenna under test to fulfill the conditions assumed in the computations. In the latter experiment, the field incident on the receiving antenna under test was not a vertically polarized plane wave. The deviation between calculation and experiment caused by this geometry is discussed in Sections II(A) and (B).

The response of the monopole probe used for measuring the transient electromagnetic field was investigated by comparing its response to the known time history of a pulsed field set up locally by TEM waves guided along an open transmission line. For this measurement, a single-wire transmission line (of wire radius 0.15 cm) is supported by a number of foamed polystyrene blocks at a height of one cm above a metal ground screen. Near the ground plane the electric field is essentially normal and may be considered a good approximation of a plane linearly polarized electromagnetic wave.

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